



# THE UNIVERSITY *of* EDINBURGH

## Edinburgh Research Explorer

### The Measure of Knowledge

**Citation for published version:**

Treanor, N 2013, 'The Measure of Knowledge' *Nous*, vol 47, no. 3, pp. 577-601. DOI: 10.1111/j.1468-0068.2011.00854.x

**Digital Object Identifier (DOI):**

[10.1111/j.1468-0068.2011.00854.x](https://doi.org/10.1111/j.1468-0068.2011.00854.x)

**Link:**

[Link to publication record in Edinburgh Research Explorer](#)

**Document Version:**

Peer reviewed version

**Published In:**

*Nous*

**Publisher Rights Statement:**

This is the Author's Accepted Manuscript of the following article: © Treanor, N. 2012, "The Measure of Knowledge", In : *Nous* 47(3) p 577-601, which has been published in final form at <http://dx.doi.org/10.1111/j.1468-0068.2011.00854.x>

**General rights**

Copyright for the publications made accessible via the Edinburgh Research Explorer is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

**Take down policy**

The University of Edinburgh has made every reasonable effort to ensure that Edinburgh Research Explorer content complies with UK legislation. If you believe that the public display of this file breaches copyright please contact [openaccess@ed.ac.uk](mailto:openaccess@ed.ac.uk) providing details, and we will remove access to the work immediately and investigate your claim.



Forthcoming in *Noûs*. Please see the final published version for quotation/citation.

### **The Measure of Knowledge<sup>1</sup>**

Nick Treanor  
University of Cambridge

We know a lot.... We have all sorts of everyday knowledge, and we have it in abundance. To doubt that would be absurd.

– David Lewis, “Elusive Knowledge”

It is a rather curious fact in philosophy that the data which are undeniable to start with are always rather vague and ambiguous. You can, for instance, say: ‘there are a number of people in this room at this moment.’ That is obviously in some sense undeniable. But when you come to try and define what this room is, and what it is for a person to be in a room, and how you are going to distinguish one person from another, and so forth, you find that what you have said is most fearfully vague and that you really do not know what you meant. That is a rather singular fact, that everything you are really sure of right off is something that you do not know the meaning of [...].

– Bertrand Russell, *The Philosophy of Logical Atomism*

I know more now than I did when I was 10 years old. This claim is, to use Russell’s expression, “obviously in some sense undeniable”. Or at least, if we set aside skeptical worries and grant that I had some knowledge when I was 10 years old and that I have some knowledge now, then it seems obviously true and unremarkable to say that I have more knowledge now than I did then.

The same is true, surely, of you: You know more now than you did when you were 10 years old.

---

<sup>1</sup> Versions of this work have been presented at the Freie Universität Amsterdam, the 2009 Bellingham Summer Philosophy Conference, the University of Geneva, a meeting of the UK Mind Network, and at the Serious Metaphysics Group, and a faculty colloquium, at Cambridge. I have benefited on each occasion from the discussion and thank in particular Don Fallis and Joshue Orozco, who commented at Bellingham, and Frank Jackson, who commented at the faculty colloquium. I am also grateful to an anonymous referee for this journal, to Duncan Pritchard and the University of Edinburgh for hosting me as a visitor while I worked on this paper, and to the Centre for Research in the Arts, Social Sciences and Humanities at Cambridge for an early career fellowship that provided me leave to finish it.

Moreover, it is not merely that these claims seem obviously true, it is that we are deeply committed to the possibility of their being true – to it being at least in principle possible for one person to know more than another, or for a person to know more at one time than she does at another. Regardless of whether you or I know more than we did when we were 10 years old, there is at least something it would be for this claim to be true.

But what would that be? What I want to press in this paper is that the claim that one person knows more than another is, to quote Russell again, something that we “do not know the meaning of”. So too for the claim that we have knowledge “in abundance”. That there is something philosophers do not understand is hardly surprising. But knowledge, and the having of more of it, is so central to philosophy that the ignorance is especially troubling here. Moreover, the issue of what it is to know more connects to central questions in the philosophy of mind, reveals and probably reduces to important metaphysical issues, and can reform our understanding of epistemic normativity.

To get to this, let me first both broaden and narrow the scope of the question. The truisms mentioned above involve simply the idea of knowing more, or of knowing a certain amount, without any domain restriction. I know more now than I did when I was 10 years old, and you, I, David Lewis and everyone else all have knowledge in abundance. But we also think it is obviously true that it is possible to know more or less, and to know a lot, about some domain or subject matter, and we are again deeply committed to this being the case. For instance, I know more about New York than I did 10 years ago. I know more about the metaphysics of mind than I do about the problem of free will. Jaegwon Kim knows more about the metaphysics of mind than I do. Here, as in the unrestricted example, we have something like the idea that the knowledge we have comes in amounts such that a person has, at a time, some amount of knowledge about a domain. We want to know, therefore, both what it is to know more in an unrestricted sense, and what it is to know more about a subject or domain.

Having broadened our scope, we can in turn narrow it. To understand what it is to know more, one has to understand what it is to know anything, period. This is a familiar problem and one I set aside here. The focus is rather on understanding the measure involved. Assume we know what knowledge is (justified true belief, warranted true belief, virtuously formed true belief, etc.). What is it to have more of *that*?

I will start by examining and arguing against a very natural approach to the measure of knowledge, one that appeals to cardinality and especially to the natural numbers. This approach generates an enumerative or aggregative model on which *how much* is a matter of *how many*. Some of the complaints are merely suggestive and hence are designed to discredit, rather than disprove, the view. But some complaints are fatal (it will be clear which are which). I then turn to the quasi-spatial notion of counterfactual distance and show how a model that appeals to distance avoids the problems that plague appeals to cardinality. But such a model faces fatal problems of its own. Reflection on what the distance model gets right and where it goes wrong motivates a third approach, which appeals not to cardinality, nor to counterfactual distance, but to similarity.<sup>2</sup> I close the paper by advocating this model and briefly discussing some of its significance for epistemic normativity. In particular, I argue that the 'trivial truths' objection to the view that truth is the goal of inquiry rests on an unstated, but false, assumption about the measure of knowledge, and suggest that a similarity model preserves truth as the aim of belief in an intuitively satisfying way.

Before I turn to these tasks, three preliminary remarks. First, I have framed the issue as one about knowledge, about the having of more of it. However, my real interest is in ignorance, in the having of less of it. Or to use a normative term, in the amelioration of it. I think the

---

<sup>2</sup> The proposal here advanced was developed independently of, but has much in common with, similarity-based accounts of verisimilitude in the philosophy of science. See especially Hilpinen 1976. Lewis (1986a, 24-27) as usual has apt insights. There are significant differences in argument, detail and consequences, and I suspect that those who have worked to develop similarity-based accounts of verisimilitude would be hesitant to embrace much of what I say. Regardless of whether they would think I am right, it should be clear that I think they are right, at least in outline if not in detail.

conceptual difference is interesting and subtle and that ignorance is a more fundamental notion. But I will by and large avoid this for now and focus in the main on knowledge, since it is a more natural way to frame most of the points.

The second preliminary remark: so far I have raised two questions: “What is it to know more?”, and “What is it to have some amount of knowledge?”. We should note, however, that the fact that there is something it is for one thing to be more F than another may not entail that there is something it is for each to be some amount of F. For instance, consider the property hardness.<sup>3</sup> It may be that hardness should be understood as fundamentally comparative: object 1 is harder than object 2 if and only if object 1 can abrade object 2 but object 2 cannot abrade object 1. Hence there could be a fact of the matter concerning whether object 1 or object 2 is harder, without there being a fact of the matter concerning how hard either object is, where this is something like some amount of hardness. I do not know whether, with knowledge, the fundamental intuition is that it is possible to know more and less than one does, or whether it is that a subject has at a time some amount of knowledge. I will, however, focus principally on what it is to know more and less, leaving open whether one knows more and less in virtue of having some amount of knowledge. For this reason, when I talk about the measure of knowledge, and in various places of needing a metric for knowledge, this should be interpreted as only asking for at least an account of what grounds an ordinal ranking of epistemic states or agents.

The final preliminary remark concerns whether there is always a fact of the matter concerning the ordinal ranking of any two subjects in terms of how much they know. Sure, you know more now than you did when you were 10 years old. But does that commit us to there being a fact of the matter concerning how, say, you and I compare? Could it not be the case that how much you know and how much I know are incomparable? The same worry applies to knowledge of domains. Some mathematician specializes in topology, while her colleague works on

---

<sup>3</sup> Thanks to Hugh Mellor for this point and the example.

set theory. Need there be a fact of the matter about whether one knows more about mathematics? Or whether the topologist knows more about topology than the set theorist knows about set theory? Or than some biologist knows about biology?

There are reasonable worries here, but we have good reason to be skeptical about the incomparability claim. Grant that we have much weaker intuitions that there must be a fact of the matter concerning whether the topologist or the set theorist knows more about mathematics. The best explanation of this is not that there is no fact of the matter, but that the case as described is such that it doesn't push us toward any of the three possibilities. In contrast, think about how much you (assuming you are no topologist) know about topology, compared to how much the set theorist knows about set theory. You know something about topology; but in this case it seems obvious that the set theorist knows more about set theory. But if so then the domains themselves are not incomparable in regard to size or magnitude. Return now to the unrestricted case: while we may have no strong views about whether you know more than I do, if I were to suddenly suffer some kind of brain damage that knocked out everything I know other than, say, what I know about my hometown and the people living there, how much you know and how much I know would not seem at all incomparable. The best explanation of this is not, I think, that states of knowledge go from comparable to incomparable as they become closer in measure – indeed, that is not even intelligible.<sup>4</sup>

### **The Measure as Cardinality or Counting**

We often speak of knowledge as if it comes in pieces. If S has a true, justified belief (or whatever), S has a piece of knowledge. Hence, one may think, how much a person knows is given by how many pieces of knowledge she has. Whoever has the most pieces wins. We can cast this view as:

S1 at  $t_1$  knows more than S2 at  $t_2$  =      there is a number  $n_1$  such that the number of S1's true justified beliefs =  $n_1$ , and there is a

---

<sup>4</sup>Thanks to Nick Denyer for good advice on how to express this point.

number  $n_2$  such that the number of S2's true justified beliefs =  $n_2$ , and  $n_1 > n_2$

This approach counts beliefs; whoever has the greatest number of beliefs that meet certain qualitative criteria (true, justified, etc.) knows the most. Similarly, presumably, on this account whoever has the greatest number of beliefs concerning some particular domain that meet those qualitative criteria knows the most about that domain:

NT<sub>2011</sub> knows more about New York than NT<sub>2001</sub> does = there is a number  $n_1$  such that the number of NT<sub>2011</sub>'s true justified beliefs about New York =  $n_1$ , and there is a number  $n_2$  such that the number of NT<sub>2001</sub>'s true justified beliefs about New York =  $n_2$ , and  $n_1 > n_2$

This domain-specific account requires refinement; chiefly, we would want to know what it is for a belief to be about a domain. But I won't dwell on that, for what I want to focus on is the idea that the quantity involved in knowing more is cardinality. This is perhaps the most natural story, but there are deep problems with it.

A first problem is that it may well be that every believing subject's beliefs are denumerably infinite and hence that the cardinality of the belief sets of any two believers is the same. The claim that we each have a denumerable infinity of beliefs will seem absurd on some views of what mental representation involves, but the issue is not straightforward and cannot be settled here. The important point is that *if* we each have a denumerable infinity of true, justified beliefs, as some theories of mental representation mandate, yet differ in how much we know, then the measure of knowledge is not cardinality. This is because (trivially) every countable infinity has the same cardinality.

The problem is familiar in a different context, since it infects Davidson's claim that charity requires that we interpret a subject such that most of her beliefs are true. Usually, Davidson casts the principle of charity in terms of number:

The nature of correct interpretation guarantees both that *a large number* of our simplest beliefs are true, and that the nature of those beliefs is known to others. (1991, 160, italics mine)

He is following Neil Wilson, who in the original formulation of the principle of charity put it in terms of numbers: We translate in a way that makes “the largest possible number of statements true.” (Wilson 1959, quoted by Quine in *Word and Object* II, §13) But Davidson is sensitive to the problem of making sense of the principle so rendered:

The basic methodological precept is, therefore, that a good theory of interpretation maximizes agreement. Or, given that sentences are infinite in number...a better word might be *optimize*. (2001a, 169. Italics his.)

Davidson’s issue, and the worry he runs into, are close to our own. In both cases, the problem is that an analysis of amount (of doxastic agreement between speaker and interpreter, or of amount of true belief) seems to founder in the face of the possibility that an infinite number of propositions fall within the content of a subject’s believing.<sup>5</sup>

The problem is decisive for the matter at hand if we shift from talking about how much one knows to how ignorant one is. This is because those who are convinced our beliefs are finite will almost certainly concede that the number of truths is infinite.<sup>6</sup> Yet if the number of truths is infinite, then each of us is ignorant of just as many truths as we ever have been. The argument is a *reductio* – since I am obviously not as ignorant as I was when I was 10 years old, and since the cardinality of the set of truths of which I am ignorant now is identical to the cardinality of the set of truths of which I was ignorant then, the metric involved is something else.

Moreover, move back now from ignorance to knowledge and consider our epistemic complements, where these are subjects who know exactly those truths that we do not. The epistemic complement of my 10-year-old self knows more than the epistemic complement of my

---

<sup>5</sup> There might be easy cases. If two subjects each have a denumerable infinity of true, justified beliefs, but the first subject’s beliefs are a proper subset of the second subject’s beliefs, then we can understand how the first has more even though both have an infinite number: the cardinality of the sets would be the same, but the second set would contain the first. But such cases are too rare to be helpful: I know more now than I did when I was 10 years old, but I knew some stuff then that I don’t know now. (Alex Oliver has made just this point in regard to measuring the ontological economy of a theory by appeal to the number of entities that exist according to the theory. (1996, 7))

<sup>6</sup> Indeed, uncountably infinite. But whether there are a countable or uncountable infinity of truths, and if the latter, which uncountable infinity, is immaterial to the point made in this paragraph.



current self. Yet if the number of truths is infinite, then the cardinality of their sets of true justified beliefs is the same. So again the measure must be something else.

So long as the number of truths is infinite, the cardinality of the sets of true justified beliefs held by my various epistemic complements does not vary, even though some know more than others. But we might insist that *we* have a finite number of beliefs, and hope that for more humble beings like us, knowing more or less will supervene on greater or lesser cardinality of the set of propositions known: if you have 235,465,443 true justified beliefs and I have 157,452,343 true justified beliefs, then you know more than I do. In other words, even if the measure of knowledge does not consist in cardinality, counting may be all that's needed to determine how much is known by a being with a finite number of beliefs. A serious problem with this, however, is that it is quite unclear whether it is right to think of beliefs as the kinds of things a person could have some number of, on the grounds that they may lack the individuation conditions that are required for the idea to be intelligible. At times this, rather than the claim that a subject's beliefs are infinite, seems to be what worries Davidson about the numeric rendering of the principle of charity:

This way of stating the position [that is, in terms of number of beliefs] can at best be taken as a hint, since there is no useful way to count beliefs, and so no clear meaning to the idea that most of a person's beliefs are true. A somewhat better way to put the point is to say there is a presumption in favor of the truth of a belief that coheres with a significant mass of belief. (2001b, 138-139)

The worry is not Davidson's alone. Consider Michael Williams's remark that:

No one has the faintest idea how *many* beliefs he has, or even how to go about counting them. This isn't just because we have so many beliefs that we wouldn't know where to begin, though this is perfectly true. Rather, we lack clear criteria for individuating beliefs – that is, saying when beliefs are the same and different – without which there is no possibility of counting. Asking how many beliefs I have is like asking how many drops of water there are in a bucket: who's to say? I believe that my dog is in the garden right now; do I *also* believe that he is not in the house, not in the basement, not in Siberia? Or are these beliefs somehow included in the original belief? (2001, 131, italics his)

As Williams words the issue, it is unclear whether his worry is epistemic or metaphysical. If the problem is merely that we do not know how to individuate beliefs, there is no threat to an enumerative approach to understanding the measure of knowledge. But I take Williams's point to be metaphysical rather than epistemic. It is not just that we do not know how to count beliefs; it is that they are not in principle countable.

The claim that beliefs are not in principle countable is remarkably common,<sup>7</sup> but it is very puzzling, and less commonly explained and defended than expressed in passing. We do talk about individual beliefs, and in the seminar room no less than in the street. Williams, after all, denies that beliefs are countable but in the same passage refers to them as "many". Moreover, surely, one wants to say, *that here is a hand* is one belief, *that here is another* is another. Why ever think, therefore, that beliefs are not in principle countable?

One might hope to support the non-countability of belief by appeal to direct intuition. Are we really comfortable saying that there is some number of beliefs we acquired yesterday, that at any given instant each of us has either an odd or even number of beliefs, that at the moment I have (for instance) 487,219,567 beliefs? To many philosophers, there is something deeply wrong with this, even if it is hard to say exactly what. At best, however, these intuitions give us reason to wonder whether belief is countable; they raise the question rather than answer it.

If believing subjects do not have some number of beliefs, it is not merely that they happen to fail to have some number of beliefs; it is that they could not have some number of beliefs. And the only way for that to be the case is if the notion of a belief is somehow defective. Not the notion of *belief* or *believing*, the notion of *a belief*. And two lines of argument for this conclusion are hinted at in the quotation from Williams above, both of which centre on the idea

---

<sup>7</sup> For instance, one often reads or hears remarks like the following: "[I]t probably makes no sense, strictly speaking, to talk of the number of things one believes....The prospects of arriving even at a principle for counting beliefs, let alone at an actual number of them, seem dim." (Stroud, 2000, 6)

that beliefs lack the individuation conditions that are required for it to be possible for a subject to have some number of beliefs.

First, one could argue that there are no criteria for belief identity, for saying when beliefs are the same or different. This is what Williams is getting at when he says that we lack criteria for "saying when beliefs are the same and different – without which there is no possibility of counting". It is an old thought, pressed hardest by Quine, who argued that we have no "standard of when to speak of propositions as identical and when as distinct." [1960, 200] That there is a serious problem with understanding the identity conditions of beliefs and propositions has been widely acknowledged, even if it has not in general dimmed our enthusiasm for employing the concepts 'a belief' and 'a proposition'. But still, Quine's demand of no entity without identity is reasonable, and it is not clear whether, as Fodor and Lepore have put it, "the notion of content identity can be made metaphysically respectable". [1999, 1]

I will not focus on this challenge, however, but on a second one, which I take to be more metaphysically interesting. One could grant that, or ignore whether, there are grounds that determine when beliefs (or propositions) are the same or different, and worry instead that there is something unintelligible about the idea of *exactly one belief*. A resolution of the issue is beyond the scope of this paper, as it involves not just the metaphysics of belief in particular but the more general and difficult question of what it is to be *one* at all.<sup>8</sup> On this point, therefore, my goal will be only to give some sense of the shape and character of the problem.

Consider Socrates's challenge to Meletus in *The Apology*:

---

<sup>8</sup> Some, like Frege, think there is no question here, on the grounds that being one is not a property: "It must strike us immediately as remarkable that every single thing should possess this property [dass jedes Ding diese Eigenschaft hätte]. It would be incomprehensible why we should still ascribe it expressly to a thing at all....It is not easy to imagine how language could have come to invent a word for a property which could not be of the slightest use for adding to the description of any object whatsoever." [Grundlagen, §29, Austin translation]

But the mistake here is twofold. First, his remarks make sense only if one assumes an ontology of objects, as it is plain he does. But that is part of the very thing at issue. And second, even if what is, is one, there should be an explanation of this. Special thanks to Henry Laycock for first alerting me to this more general question.

Does any man, Meletus, believe in human activities who does not believe in humans?....Does any man who does not believe in horses believe in horsemen's activities? Or in flute-playing activities but not in flute-players? No, my good sir, no man could. (27b)

Socrates is pointing to necessary connections between belief states: believing something at least sometimes metaphysically necessitates believing other things as well. This has become familiar in a strengthened form as the claim that it is impossible to believe anything without believing a great deal more. Wittgenstein put it with typical flair:

141. When we first begin to *believe* anything, what we believe is not a single proposition but a whole system of propositions. (Light dawns gradually over the whole.) (1969, 21)

To have an example at hand, consider the belief that higher education deserves public funding. Very plausibly, having this belief involves believing a great deal more about education, public funding, desert, and so on, where to believe a great deal more about education, public funding, desert, and so on in turn involves believing a great deal more about various other things. The idea is that someone who fails to have these other beliefs could not even be said to understand what it is for higher education to deserve public funding. But if they do not even understand what it would be for that to be true, they could scarcely think that it is true. Believing that higher education deserves public funding involves some minimal grasp of the truth-conditions of that proposition, and this in turn involves further belief.

Many are the fans of such holism, many the critics. What has not been adequately explored are its consequences for the intelligibility of the notion of an individual belief. Or more precisely, since the notion of an individual belief isn't going to go away, what has not been adequately explored is whether this notion can be coherently understood in the way it needs to be for it to be the case that a subject's believing has a cardinality.

If the driving idea of semantic or belief holism were merely that the existence of any belief entails the existence of a great many more beliefs, there would be no threat to the intelligibility of individual beliefs, any more than the fact that if anyone is a gang member then

lots of other people are threatens the intelligibility of individual gang members. But I take holism to be more the claim that believing that, for instance, higher education deserves public funding *involves*, not merely *requires*, believing other things as well. It is tempting to resort to a class of familiar dependence relations, to say for instance that the belief that higher education deserves public funding is constituted by or in some sense contains various other beliefs. Or to say that what it is to believe that higher education deserves public funding is more than merely to be in a state that necessitates that one is also, distinctly, in the state of believing various other things, but is rather to be in a state that is, in part, the believing of various other things. But these ways of putting it, in terms of constitution, containment or parthood, are not right, for they suggest either mereological relations or (at least) asymmetric dependence. What semantic holism suggests is something more seamless, where beliefs bleed into one another. Even that is not the right way of putting it, since that makes it sound as if there are individual beliefs with fuzzy, bleeding edges. The idea instead is that believing takes chunks of propositional space as its object; we have bodies of *belief*, denoted by a mass noun, rather than beliefs, denoted by a count noun. The difference between thinking of holism as involving asymmetric dependence or parthood and this alternative picture is important, because if beliefs merely contain or depend upon other beliefs then counting is still possible. What is at issue is the very different question of whether beliefs are properly understood as individuals at all.

David Braddon-Mitchell and Frank Jackson have sympathetically explored this alternative picture, although (as far as I know) not in a way that engages with the underlying, more general metaphysical issues. In a passage that effectively captures our discussion of holism so far, they remark:

What would it be like to believe that there's milk in the refrigerator, and nothing else? It seems as impossible as having money without the social and economic circumstances that give sense to something being money. To believe that there is milk in the refrigerator, you have to have enough by way of belief to count as understanding what milk is, what a

refrigerator is, and what it is for one thing to be inside another. It takes a lot of belief to be any amount of belief. [2007, 196]

The key idea is that it is not merely that it takes a lot of belief to *have* any amount of belief, it takes a lot of belief to *be* any amount of belief. If this is right, then, necessarily, whenever we're talking about what we ordinarily call an individual belief (that here is a hand, that higher education deserves public funding, that there is milk in the refrigerator) we are talking about a lot of belief. Each is one belief, sure, if we must; but here 'individual belief' is something like 'a sentence-shaped portion of belief, *the stuff*'. And the problem is that when we want to know how much someone believes or knows, we want to know the quantity of belief the stuff, not the number of sentence-shaped portions of it.<sup>9</sup> What we need is a measure on the stuff, and 'sentence-shaped portion' fails to provide one.

I have presented this line of argument as resting on semantic holism, but it may be possible to develop a substantially similar argument without this as an explicit commitment.

The first premise is that the extension of a concept is countable (and finite) only if a certain condition is met:

The concept "letters in the word 'three'" isolates the *t* from the *h*, the *h* from the *r*, and so on. The concept "syllables in the word 'three'" picks out the word as a whole, and as indivisible in the sense that no part of it falls any longer under that same concept. Not all concepts possess this quality. We can, for example, divide up something falling under the concept "red" into parts in a variety of ways, without the parts thereby ceasing to fall under the same concept "red". To a concept of this kind no finite number will belong. The proposition asserting that units are isolated and indivisible can, accordingly, be formulated as follows:

Only a concept which isolates what falls under it in a definite manner, and which does not permit any arbitrary division into parts, can be a unit relative to a finite number. (Grundlagen, §54)

---

<sup>9</sup> Braddon-Mitchell and Jackson "explain away" the intuition that "there are specific individual beliefs and desires". In their view, "there is your picture of how things are, and your picture of how you want them to be". [233-236]

There is a lot packed into this, and Frege's wording may not be wholly satisfactory. Geach provides a helpful elaboration which taken jointly with the passage from Frege should give us a serviceable enough idea of the condition for present purposes:

[O]nly in connection with some terms can the question be asked how many so-and-so's there are. For example, although we have the phrase 'the seven seas', nobody could set out to determine how many seas there are; the term "sea" does not determine any division of the water area in the world into seas the way that the term "letter" (in the typographic senses) does determine a division of the printed matter in the world into letters.<sup>10</sup> [63]

The second premise is that belief does not meet this condition. Think of the concept "believed by NT". What I believe, at least very plausibly, permits arbitrary division into parts, and those parts are not, again at least very plausibly, isolated in the sense of discrete. To be sure, we can nominally specify these divisions – I believe that here is a hand, that higher education deserves public funding, that there is milk in the refrigerator, and so on. But our ability to nominally specify divisions in what I believe is not all that's needed for what I believe to be countable; it must also be the case that there exists a privileged set of divisions such that they jointly specify all, only, and without overlap what I believe.

Elsewhere Frege, with something very different in mind, provides a nice example of the problem:

In the sentence: 'Because ice is less dense than water, it floats on water' we have

- (1) Ice is less dense than water
- (2) If anything is less dense than water, it floats on water;
- (3) Ice floats on water.

The third thought, however, need not be explicitly introduced, since it is contained in the remaining two. On the other hand, neither the first and third nor the second and third combined would furnish the sense of our sentence. [1892, 76-77]

We have here four thoughts, all of which I believe. In one sense they are distinct thoughts; each is such that it is not identical to any of the others. But they are not distinct in the sense that if this

---

<sup>10</sup> Geach also makes clear (64) why the issue of countability is distinct from that of identity. That is, there can be criteria of identity without criteria of countability. "But it is not necessary, in order that 'the same A' shall make sense, for the question 'How many As?' to make sense; we can speak of the same gold...but 'How many golds?' does not make sense".

were the totality of what I believed, I would believe exactly four things. On the face of it, the concept 'believed by NT' permits arbitrary division of its extension into parts, and fails to isolate what falls under it in a definite manner.

It may be tempting to think the foregoing discussion concerns merely the vehicle of believing rather than what is believed. That is, one may want to object: "Fine, you can't cut up a person's belief state and count, but that's irrelevant. What matters is whether some number of propositions fall within the content of a person's believing. If, say, 987 do, then the person has 987 beliefs." This objection misses the point, however: the discussion of the previous section concerns content, not vehicle.

The same issue comes up in a debate between Lewis and Stalnaker. In "Reduction of Mind", Lewis describes a map-like theory of belief as an alternative to the language-of-thought hypothesis:

A serious issue...concerns the relation between the whole and the parts of a representation. Suppose I have a piece of paper according to which, *inter alia*, Collingwood is east of Fitzroy. Can I tear the paper up so that I get one snippet that has exactly the content that Collingwood is east of Fitzroy, nothing more and nothing less? If the paper is covered in writing, maybe I can; for maybe 'Collingwood is east of Fitzroy' is one of the sentences written there. But if the paper is a map, any snippet according to which Collingwood is east of Fitzroy will be a snippet according to which more is true besides. For instance, I see no way to lose the information that they are adjacent, and that a street runs along the border. And I see no way to lose all information about their size and shape....

Mental representation is language-like to the extent that parts of the content are the content of parts of the representation. If our beliefs are 'a map...by which we steer', as Ramsey said, then they are to that extent not language-like. And to that extent, also, it is misleading to speak in the plural of beliefs. What is one belief? No snippet of a map is big enough that, determinately, something is true according to it, and also small enough that, determinately, nothing is true according to any smaller part of it. If mental representation is map-like...then 'beliefs' is a bogus plural. You have beliefs the way you have the blues, or the mumps, or the shivers. (1999, 311, *italic his*)

At first glance, the argument is compelling: if belief is map-like, then although there is a whole representation, there is no exhaustive decomposition of this whole representation into parts – there is no way to cut the whole representation into snippets such that each snippet is exactly one



belief. But that is beside the point, since Lewis appears to be talking about the vehicle of representation rather than about the content of representation. Stalnaker makes just this objection in “Lewis on Intentionality”:

Lewis suggests that if the way information is represented is holistic – map-like, or hologram-like, rather than sentence-like – then plural propositional attitude terms such as ‘beliefs’ are ‘bogus plurals’: ‘You have beliefs the way you have the blues, or the mumps, or the shivers’. But this is not right, since whatever the nature of the vehicle or vehicles of mental representation, the plural noun ‘beliefs’ does not refer to that vehicle, or to those vehicles. What it refers to is the *contents* of a representation – to the propositions that are believed. And even if there is a single map-like internal structure in a believer that make[s] it the case that she has the beliefs that she has, that structure will determine a plurality of propositions that are believed.....[W]hatever the character of what is going on in a person’s head when she has beliefs, these goings on should not be confused with what the person believes. (2004, 208, italics his)

To a point, Stalnaker is right. What is at issue when we are wondering whether a subject’s beliefs are countable is whether the content of her believing is countable, not whether the vehicle by which she believes what she believes can be decomposed into smaller entities that correspond one-to-one with what is believed. But he is right only to a point because what Lewis proposes may be true of the vehicle is very plausibly true of the content. What is one content, or one bit of content? What is a propositional content that is exactly one proposition – nothing more and nothing less? We want to answer with a sentence and say the sentence picks out or expresses exactly one thing – that higher education deserves public funding, that ice floats on water because it is less dense than water, that Collingwood is east of Fitzroy. But for any of these propositions to be true is for much to be true – since it takes a lot of truth to be any amount of truth.

We should not underestimate the difficulties with articulating this view in a satisfactory way. For now, we should take it as helping us focus on the heart of the issue. The question of how many true justified beliefs a person has is really the question of how many facts she knows, of how many truths lie within her ken. Part of the difficulty with understanding what it is to have some number of true justified beliefs is the difficulty in understanding how to draw boundaries around what falls within someone’s ken (that the dog is in the yard, or also that the dog is not on

Jupiter?). But the difficulty is also that of understanding how to draw boundaries within what is known, that is, of understanding what it is for there to be some number of truths or propositions. Does the world divide into facts? Are there some number of truths out there, lying in wait to be scooped up in our doxastic nets?

We need not explore these questions further here since a problem remains, even if there are some number of truths, and even if each of us knows some finite number of them. Grant that a person's knowledge decomposes into knowledge of some finite number of truths and let us suppose that you know 235,465,443 and that I know 157,452,343; this will entail that you know more than I do only on the substantial assumption that what is true divides into truths *of the same size*. It is hardly clear what this means (that is the very thing at issue!). But what is clear is that the fact that the extension of a concept consists of, or can be decomposed exhaustively into, a finite number of individuals does not entail that the correct measure on the extension is cardinality. For example, consider the theological concept of sinning and assume it is in principle possible to count token instances of sinning. What matters on Judgement Day is surely not how many sins a person has committed, but how much sin. You *can* count sins, but that is no way to measure sinning. This general point is relevant because there is intuitive reason to think that the truths we know are, like the sins we commit, not all the same size.

There are two questions to distinguish. The first is whether ordinary 'individual truths', of the kind we express in everyday natural language sentences, differ in size. The second is whether, if what we know decomposes into some finite number of truths, these bottom-level truths differ in size. The answer to the first question is obviously yes: compare the truth that ice floats on water with the truth that ice floats on water because it is less dense than water. Less obvious, but nonetheless intuitive, is a different sort of example. Suppose it is true that the universe underwent massive exponential expansion shortly after the Big Bang, and that the number of threads in the shirt I am wearing is even. Is there no sense in which in knowing the

first truth one knows more than in knowing the second truth? The idea is not that one knows something more interesting or more prudentially valuable; whether that is the case turns on who you are and what you want. The suggestion instead is that knowing the former truth increases one's epistemic contact with the world much more than does knowing the second truth. Or consider as another example the difference between knowing that some object is green and knowing that some object is grue. If I know that there is a green object under my desk, I seem to know more than I do if I know that there is a grue object under my desk. Or compare knowing that there are electrons to knowing that something is called an 'electron' in English.

The answer to the first question is obvious, but not what is at issue. What we want to know is whether differences in the size of truths expressed by ordinary language sentences reduce to differences in the cardinality of the metaphysically basic veritic elements into which, we are supposing, such truths can be decomposed. This strategy suggests itself immediately with the first example, involving truths about the floating of ice, even if things turn to slush a cut or two in. But one might also hope this strategy will work with the second set of examples: maybe one knows more when one knows that the universe underwent massive exponential expansion shortly after the Big Bang than when one knows that the number of threads in the shirt I am wearing is even, but this is only because both pieces of knowledge are compounds of basic elements, and the former compound has more (i.e., on this reading, a greater number of) basic elements than the latter.

This is the second question, and it cannot be answered as directly as the first. For one thing, we don't have the slightest idea of what the bottom-level truths would look like, as it were. Certainly we can bring no examples to hand: that is the force of the preceding section on holism. But if we grant that such truths exist, in the sense that what is expressed by an ordinary true sentence of natural language decomposes into some finite number of them, then we need a way to assess whether they differ in size. This we can do: What we need to do is imagine a possible

situation that meets three conditions. First, it is one in which one subject knows more than another. Since we do not know what it is for one subject to know more than another, we will have to be guided by intuition here. Second, it is one in which the subject who knows more does not know some greater number of bottom-level, or atomic, truths. Since we do not know what such truths look like, or how many are in this true sentence and how many in that, we will have to be careful to ensure, by stipulation if necessary, that every atomic truth known by the subject who knows more is matched by at least one atomic truth known by the subject who knows less. And third, it is one in which each subject knows some finite number of atomic truths. The compossibility of the first two conditions demonstrates the measure of knowledge does not consist in the cardinality of atomic truths known. But the third condition ensures the argument speaks to the broader point at issue, which is whether, even if the measure of knowledge does not consist in cardinality, counting may give the measure for beings with a finite number of beliefs.

Compare Smith and Jones, who come into the world as ignorant as the rest of us, but who are blessed with extraordinary intellects, phenomenal memory, astute perception, and so on. Smith sets out to travel the world and in the fullness of time comes to know everything in the current, 30-volume *Encyclopedia Britannica*. (That is, we are assuming everything in the *EB* is true and that he comes to know it all.) Jones, meanwhile, has been trapped his whole life in a small, windowless room, with nothing to read, no one who visits, no communicative pipeline at all to the world outside his room. But the room has a small and crude tape measure and so Jones spends all his time pulling it out a certain distance, remembering what number it bears, and then repeating this process. When Smith discovers the strong nuclear force, Jones discovers that on the 18,756,391 time he pulls out the tape measure it reads somewhere between 55 and 56 inches. When Smith discovers the chemical process that converts carbon dioxide into organic compounds using energy from sunlight, Jones discovers that on the 25,234,344 time he pulls out the tape measure it reads around 43 inches. When Smith discovers that the universe contains many

billions of galaxies, Jones discovers that on the 32,535,113 time he pulls out the tape measure, it reads – lo – somewhere between 55 and 56 inches again. And so on. Since we don't know how many atomic truths Smith has learned, let us stipulate that Jones works quickly enough that for every atomic truth Smith learns, Jones learns one of his tape-measure-pulling truths. (I assume that each of these truths is at least one atomic truth; surely, if the truths expressed by ordinary sentences decompose into atomic truths, each tape-measure truth decomposes into at least a handful.) Finally, let us agree that the number of atomic truths known by Smith is finite, and so too for Jones. If this were not the case, then since what they know is not different in kind to what you and I know, it will turn out that you and I also know an infinite number of atomic truths, and the whole issue will be moot.

In this situation, it seems to me, there is no serious question that Smith knows more than Jones. This claim will not be intelligible if you import into the meaning of “knows more” the theoretical claim that “knows more” = “knows some greater number of truths”. The idea is to back away from such theoretical commitments when you think about the situation. Smith knows a great deal; that can't be doubted. But Jones's cognitive labours have got him almost nowhere. He has learned unspeakably many truths but remains for all that massively, almost wholly, ignorant.

Let's try a different situation, this time restricting ourselves to knowledge of a domain. Smith knows everything in the *Encyclopedia Britannica* about apples, whereas Jones knows a zillion truths of the following form: he has a proper name A that in fact denotes apples, and he knows  $\neg(A = n_1)$ ,  $\neg(A = n_2)$ ,  $\neg(A = n_n)$ , where  $n_1...n_n$  are points on some one inch line. In this case, it seems clear he is massively ignorant, almost wholly ignorant, of apples, even though he knows as many truths about apples as you please. Why think this? Because he hasn't a clue what an apple is. Smith knows far fewer truths about apples, but for all that vastly more.<sup>11</sup>

---

<sup>11</sup> One might object that Jones can't be said to know anything about apples (and hence not even that apples are not identical to this point, not identical to that point, etc.) unless he has some minimal grasp of what an

I said there is no serious question that Smith knows more; but there is a serious question of why Smith knows more. A first thought might be that (to discuss the first example) Smith can tell the difference, or discern, his world from more worlds. But this is not right, since there are just as many worlds compatible with what Smith knows as compatible with what Jones knows (infinitely many). But the thought is in the right area, since what it points toward is the idea that reality is more like how Smith takes it to be than it is like how Jones takes it to be. To be sure, neither is wrong about anything – that is built into the example. But the world as Smith takes it to be is much more like the world as it is than is Jones's world. So too in the example restricted to the domain of apples. There are just as many non-apple possibilities that Smith cannot rule out as apples as there are that Jones cannot rule out as apples (infinitely many). But the things that Smith can't rule out as apples are much more like apples than the things that Jones can't rule out as apples. For all Jones knows, he himself could be an apple, and anyone whose picture of apples doesn't rule out that he himself is an apple can hardly be said to know very much about apples.

What is at issue is again whether cardinality exhausts the structure of what is true. We already have a decisive argument against this (the argument that appeals to my various epistemic complements), but it is instructive to look at David Lewis's remarks on abundant properties, for abundant properties are (simplifying a little) just those picked out by well-formed predicates of a possible language.<sup>12</sup>

---

apple is, and this isn't supplied merely by having a term that, thanks to the right causal connection, picks them out. We might take this objection to prove the point, but to avoid arguing that here, we can modify the example to give Jones whatever minimal grasp of apples is required for him to have apples as a possible object of thought. Let both Smith and Jones begin with this minimal grasp, and let Jones learn a zillion truths of the form 'no apple has ever had its centre of gravity at  $n_1$ ', etc., and let Smith learn everything in the *Encyclopedia Britannica* about apples. Smith will learn tons about apples, Jones almost nothing. Or to put the point another way, at the end of this process Smith knows much more about apples than Jones does.

<sup>12</sup> "The abundant properties may be as extrinsic, as gruesomely gerrymandered, as miscellaneously disjunctive, as you please.... There is one of them for any condition we could write down, even if we could write at infinite length and even if we could name all those things that must remain nameless because they fall outside our acquaintance." [1986a, 59-60]

Because properties are so abundant, they are indiscriminating. Any two things share infinitely many properties, and fail to share infinitely many others. That is so whether the two things are perfect duplicates or utterly dissimilar. Thus properties do nothing to capture facts of resemblance. That is work more suited to the sparse universals. Likewise, properties do nothing to capture the causal powers of things. Almost all properties are causally irrelevant, and there is nothing to make the relevant ones stand out from the crowd. Properties carve reality at the joints – and everywhere else as well. [1983, 346]

We should agree with Lewis that sharing (merely) abundant properties does not make for similarity. But when we reflect on the fact that such properties are the semantic values of predicates, we should note the epistemic correlate: that knowing (merely) abundant truths about some entity does not help us know that entity. Here is an analogy: Suppose I want to duplicate some apple and to do this I make a list of its abundant properties and then build an object that shares a great many, perhaps infinitely many, of those properties. If the object I build is an apple, then great, I may have duplicated the apple, or at least have come close to doing so. But I could have built Mount Everest – since it too shares many, indeed infinitely many, properties with the apple I started with. But clearly I would not have come anywhere close to duplicating an apple by building Mount Everest. Part of the problem would be the fact that I would have gotten a lot wrong (apples are not covered year-round in snow and ice, for instance). But as much of the problem is that I would have gotten almost nothing right (even though I would have gotten an infinite number of properties right). In exactly this way, if I aim in cognition to duplicate the world – not literally, but in a representation – then merely building a representation that has a content that shares many, even infinitely many, properties with the world is no guarantee that I have come anywhere close to duplicating the world. I may have – but it depends not merely on *how many* properties I duplicate but on *which* properties I duplicate. An apple shares infinitely many properties with the actual world and to each of these properties there corresponds some well-formed predicate in a possible language. But if I learned that all those predicates were true of reality (that the world had those properties) I would have learned very little about it. For instance,

I could not tell the difference between the actual world and an apple. Whatever it is that makes for greater or lesser knowledge (or greater and lesser ignorance), it is not exhausted by cardinality.

To sum up the problems with understanding the measure of knowledge by appeal to cardinality or counting: First, if each of us has an infinite number of beliefs, as some theories of mental representation allege, or if there are an infinite number of truths, as most philosophers would be inclined to grant, then the measure of knowledge does not consist in cardinality. Second, even the much weaker view that knowing more and less supervenes on greater and lesser finite cardinality of the set of truths known faces a serious challenge, since it may not be intelligible that we know some number of truths at all. Third, even if there are some number of truths, and even if we restrict ourselves to subjects who know some finite number of them, we could not determine how much a person knows by counting the truths that she believes, since cardinality does not exhaust the structure of what is true. Let us turn, therefore, to the second approach to the measure of knowledge, which appeals not to cardinality but to counterfactual distance between worlds.

### **The Measure as Counterfactual Distance**

We have ways of thinking about quantity that don't involve numbers. A large pizza is bigger than a small pizza. We can think of this partly in terms of number – the large pizza is some number of square inches, the small pizza is some number of square inches, and the first number > the second number. But we need not, and surely no hungry child does. Similarly, when cave people distinguished between big caves and little caves, they were not counting. Moreover, in neither case can we think of the size solely in terms of number. What is involved is a spatial notion; bigger things are stretched out further, or take up more space. It is precisely the spatial extendedness of non-discrete stuff such as soup that allows us to annex a number to a quantity (cup, teaspoon, decilitre) as a way to describe or capture quantitative differences.



It is interesting to note, therefore, that when thinking about belief we often employ spatial notions. Perhaps the most familiar example is talk of beliefs forming a web, which construes our believing as either a two-dimensional plane or as a three-dimensional volume, both of which are extended regions. Another familiar example is talk of beliefs constraining epistemic space:

[T]he space of scenarios constitutes my *epistemic space*: the space of specific epistemic possibilities that are open to me a priori. If I had no empirical beliefs, all of epistemic space would be open to me. As I acquire empirical beliefs, my epistemic space is narrowed down. Any given belief will typically *divide* epistemic space into those epistemic possibilities that it endorses and those that it excludes. (Chalmers 2002, 610, italics his)

On this picture, the more one believes or knows, the smaller one's doxastic or epistemic space.

This spatial, or quasi-spatial, way of thinking about belief construes the content of our believing as a volume – as one comes to believe more, one's doxastic space shrinks. Might this, therefore, be a way we could think about the measure of knowledge without relying on numerical notions?

The problem would seem to be that although belief content is here thought of in spatial terms, the measurement problem remains. How much does one's epistemic space shrink when one comes to know, for instance, that higher education deserves public funding? Does a believer's epistemic space shrink in volume by the same amount with each new belief, or do some beliefs make it shrink more (and if so, why)? How do we compare the size of the epistemic space of two believers? What we need is not merely a way of thinking of belief in spatial, or quasi-spatial, terms, we need a metric of this space.

One place we might look for such a metric is in familiar talk of counterfactual distance between worlds. Here, amount is construed in spatial, or quasi-spatial, terms: Possible world A is more different in some respect from possible world B than it is from possible world C if and only if the distance between A and B is greater than the distance between A and C. There are several ways such a model could be developed. Just to have an illustration at hand, one might say that

how much a subject knows is given by the distance between the actual world and her furthest away epistemic world, where an epistemic world is a world compatible with her knowledge:

S1 knows more than S2 =	S2's most remote epistemic world is further away from the actual world than S1's most remote epistemic world.
Epistemic World of S =	W is an epistemic world of subject S iff W is such that for all p, if S knows p then p is true at W.
Most remote EW of S =	W is S's most remote epistemic world iff W is an epistemic world of S and no epistemic world of S is further from the actual world than W.

A virtue of this kind of account is that numbers drop out of the picture – quantity is given by distance rather than numbers. But a further virtue of this approach is that it accommodates the fact that knowing some propositions contributes more to how much one knows than knowing others. This is because distance between worlds is a function not merely of how many things are different between the worlds in question, whatever that might mean, if indeed it means anything at all, but of what those differences are. Some differences count for more. Some differences are bigger differences.

So, for instance, imagine two world-mates who share a belief system at a time, at least to the degree this is possible given that they are two believers rather than one (ignore *de se* beliefs, various other indexicals, etc.), and who have identical epistemic justification or warrant for what they believe. They know the same amount, and on the account above the amount they each know is given by the distance between the actual world and their furthest away epistemic world. Suppose further that neither subject has any belief one way or the other concerning whether the number of threads in my shirt is even, and no belief one way or the other concerning whether there are electrons. Now add to the first believer a justified true belief that the number of threads in my shirt is even. Prior to adding this belief, the subject's furthest away epistemic world was one in which the number of threads in my shirt is odd. That is because it is further away than a

possible world that is identical except for the fact that the number of threads in my shirt is even. With the belief concerning the number of threads in my shirt being even added, therefore, the subject's furthest away epistemic world jumps a little closer. Not much closer, but closer nonetheless. But now imagine adding to the other subject a justified true belief that there are electrons. That subject's furthest away epistemic world jumps significantly closer to the actual world, since a world in which there are no electrons is much further away from the actual world than a world in which there are, holding everything else as much as possible the same. In other words, whether or not there are electrons is a big difference, whereas whether the number of threads in my shirt is even or odd is a little difference. Hence this approach accounts for intuitive judgments that one knows more by knowing, for instance, that there are electrons than by knowing that the number of threads in my shirt is even. You know more by knowing the first truth because in knowing it you greatly reduce the distance between the world as it is and the world as, for all you know, it could be.

Whatever the virtues of an account like this, it would face a host of familiar problems. For instance, it may not have the resources to explain what it is for one person to know more mathematics, or metaphysics, or morality, or about anything else where necessary truth is involved. This is because necessary truths are true in every possible world and thus believing, or failing to believe, a necessary truth would make no difference to the distance between the actual world and one's furthest away epistemic world. And an account like this seems committed to the very odd consequence that once one knows  $p$ , coming to know whatever propositions  $p$  entails adds nothing to how much one knows. This is because as soon as one knows  $p$  the distance between one's furthest away epistemic world and the actual world contracts to the distance at which  $p$  and everything it entails is true. What is at issue is not merely logical entailment in the sense of idealized a priori derivability. For suppose we knew all and only the microphysical facts. Much that uncontroversially supervenes on the microphysical is such that, very plausibly, we

would come to know more if we were to come to know it. Examples include all the truths of chemistry, biology, geology and the other special sciences, as well as such mundane observations as that there are a number of people in this room at the moment.

But familiar problems aside, there is a more basic problem with this kind of approach. The proposal identifies amount of knowledge with counterfactual distance, but this merely pushes the bubble elsewhere under the rug.<sup>13</sup> For there just is no clear, well-worked-out account of what such distance is – what that metaphor means or amounts to. There are in fact two problems here. First, we do not really have any good accounts of what criteria determine distance. There are incomplete gestures,<sup>14</sup> but these are as much an expression of what needs to be explained as they are an explanation of it. And second, accounts that have been adumbrated appeal to notions of quantity that are as opaque as, and perhaps even involve, that which is involved in knowing more. I here have in mind the notion of amount of similarity. The point may be hard to see, since it is commonly said that counterfactual distance is overall similarity. But that is elision; counterfactual distance is overall similarity under some system of weights and measures of similarity in particular respects. Fledgling accounts of counterfactual distance rest on, rather than explicate, the notion of similarity.

In light of these problems, if our theory of the metric involved in knowledge reduces it to the metric involved in counterfactual distance, one might well worry that our efforts at theorizing haven't brought us any closer to understanding that which we are theorizing about. Note, though, that there would still be virtues to this kind of account, even if the bubble remained. For it would assimilate one problem, that of how to understand how much a subject knows, to another problem, that of how to understand counterfactual distance, which we want to figure out for other reasons. It would not solve any problems, but it would leave us with one fewer.

---

<sup>13</sup> Thanks to Kelly Trogon for suggesting this turn of phrase.

<sup>14</sup> e.g., Lewis 1979.

We have then two classes of objection to the quasi-spatial approach – what I described as familiar problems, which point to peculiar and implausible consequences of a distance-between-worlds model, and what we can think of as a foundational objection, which insists the approach does not solve the problem but merely relocates it. Reflection on both strands of objection can lead us to a third approach, which appeals not to cardinality, nor to counterfactual distance, but to *similarity*. This approach will avoid at least many of the problems that threaten the cardinality and distance models. And it will preserve what the distance model gets right, which is that knowing more isn't a matter of increasing the cardinality of the set of truths known, but should instead be understood by appeal to the notion of reducing difference between the world and the world as it is represented as being. It will also expand the scope of the theoretical assimilation that was offered as a virtue of the distance model. The bubble will remain, but it will have been relocated from one centred just on the measure of knowledge to one that is centred on a relation that features widely, centrally, and probably indispensably throughout philosophy and the sciences.

### **The Measure as Similarity**

A central weakness of the counterfactual distance proposal is that it appeals to a relation holding between possible worlds, which are maximal, in the sense of settling every question. This is what generates the problem of understanding how we come to know more when we come to know truths entailed by, or that supervene on, what we already know, and the problem that knowing necessary truths adds nothing to how much one knows. We can, however, preserve the virtues of the distance proposal if we construe 'amount known' by appeal to a distance-like relation that holds between entities when at least one is sub-maximal (not a complete way or specification of how things could be). To satisfy this criterion, one might appeal to “partial worlds” and take there to be a distance relation between partial worlds, just as there is between worlds, or between partial

worlds and the actual world. But this is to look for smoke when we already have the fire. We have a relation and relata that are familiar, in broad use, and probably indispensable: the relation of similarity and the relata of representations and what they represent.

Similarity holds sub-maximally. This point is not demonstrated by pointing to two apples and an orange and remarking that the two apples are more similar to one another than either is to the orange. For although neither of the apples, nor the orange, is a world, each is a maximal way that a thing could be. That is why although no actual apple or orange is a world, any apple or orange could be; a lonely (in the Kim/Langton/Lewis sense) apple or orange would settle every question. Or, to put it another way, for every apple, every orange, and every property, the apple or orange either has that property or lacks it. But the point is demonstrated by pointing to the similarity relation that holds between a representation and what it represents. Here I do not mean between a vehicle of representation and what it represents – I mean between the content of a representation and what it is a representation of. Consider a painting of Genghis Kahn. It is similar to him in certain ways. It is made of carbon, came into existence after Socrates did, has never been to Jupiter, and so on. But it (the painting) is not sub-maximal – for every property, it either has it or lacks it. But the painting's representation of Genghis Kahn represents an incomplete way that an entity could be. For instance, it does not specify what fundamental particles compose him, or even whether he is composed of fundamental particles. It leaves open the year of his birth, whom he loves, what he regrets, whether he has ever tasted cheese. And so on. Nothing could be just as Genghis Kahn is represented as being. But for all that, there is a similarity relation between Genghis Kahn and Genghis Kahn as he is represented as being.<sup>15</sup>

The point here is made by appeal to paintings, but it is a general point about representation. The government issues an edict and the people riot. Newspapers report on the

---

<sup>15</sup> This should be read not that Genghis Kahn is represented as being such that he has indeterminate properties, but rather that Genghis Kahn is represented such that it is indeterminate which properties he has.

uprising, and variously represent the event as being thus and so. The reports are, of course, incomplete; who would want them to be otherwise? But for all that, they may differ in terms of how similar the riot is to how they represent it as being.

Because similarity holds sub-maximally, an account of the measure of knowledge that construes it as the similarity between the world and the representation of it afforded by a subject's knowledge avoids what I called familiar problems that plague the counterfactual distance proposal. It is also immune to the problems that threaten an appeal to cardinality. That two representations each share infinite properties with what they represent is no bar to the one representation being more similar to the represented entity than the other, as Lewis's remark about abundant properties reminds us. And we can endorse a similarity model of the measure of knowledge without committing to the claim that what is true decomposes into isolated, non-arbitrary countables, just as we can say that two oranges are more similar to each other than they are to some apple without committing to the claim that each object's being as it is decomposes into the instantiation of some cardinal number of properties. A similarity model also accommodates the fact that knowing some truths contributes more to how much one knows than knowing other truths: How similar some manufactured object is to an apple depends not merely on how many abundant properties it has in common with the apple, assuming that is even an intelligible claim, but on what those properties are, and for just this reason – since abundant properties are the semantic values of predicates – how similar one's knowing representation of some object is to the object itself depends not merely on how many truths one knows about it but on what those truths are.

An account that appeals to similarity avoids these problems while expanding the theoretical gain of the counterfactual distance proposal. That is, the distance model did not purport to solve the problem but rather reduced it to another problem, that of understanding distance between worlds. We express the similarity relation that holds between a representation

and what it represents in various ways. All are opaque: likeness, fit, fidelity, accuracy, resemblance, correspondence. But the core notion is that of similarity and this notion, though not well understood, is one that we are stuck with already. It features in some important way in every philosophical project. But it is not a philosopher's invention: it seems essential to our capacity to generalize, to reason by analogy, to form and individuate concepts and categories. If we construe the measure of knowledge as the similarity between the world and one's knowing representation of it, we have not solved the problem, since similarity is not well understood. But we will have reduced the problem to one that is deep and ubiquitous.

What I advocate, therefore, is not an account of the measure of knowledge, where this is understood as a full story of what the measure of knowledge consists in. Rather, it is a reductive move: the measure involved in knowing more is the measure involved in similarity. To be sure, many questions remain: Is similarity primitive? Or is a reduction of similarity, perhaps to structural identities, possible? (We know that no reduction to number of properties in common will be forthcoming.) The proposal also requires, at least if we are to preserve an ordinary, interest- and context-independent sense of knowing more and being less ignorant, that famous attacks on the intelligibility of objective overall similarity be defeated. Nonetheless, we can affirm these as unsettled issues while still holding that until we have a plausible account of what similarity could reduce to, we have found the right place to rest.

The proposal, it should be said, is as much a retreat as an advance. Let me adapt a remark that Lewis makes in a closely related context. In his postscript to "Counterfactual Dependence and Time's Arrow", Lewis reiterates his rejection of what he calls egalitarianism:

It is widely thought that *every* shared property, in the most inclusive possible sense of that word, is *prima facie* a respect of similarity: that things can be similar in respect of satisfying the same miscellaneously disjunctive formula, or in respect of belonging to the same utterly miscellaneous class. If so, then there's little to be said about comparative similarity. Any two things, be they two peas in a pod or be they a raven and a writing-desk, are alike in infinitely many respects and unlike in equally many....[But it] just isn't



so that all properties (in the most inclusive sense) are equally respects of similarity.  
[1986b: 53, italics his]

His rejection and what he rejects have close parallels in this paper: although one might hope that knowing more is a matter of accumulating a greater number of true, justified beliefs, this just isn't so. Even if, as is doubtful, a subject's believing at a time has a cardinality, not all truths make an equal contribution to one's epistemic contact with the world. This is because, or is at least intimately tied up with the fact that, not all properties, understood in the inclusive sense as the semantic values of well-formed predicates in a possible language, are equally respects of similarity.

Lewis then goes on to say:

Once we reject egalitarianism, what shall we put in its place? An analysis, somehow, of the difference between those properties that are respects of similarity and those that aren't? A primitive distinction? A distinction built into our ontology, in the form of a denial of the very existence of the alleged properties that aren't respects of similarity? A fair question; but one it is risky to take up, lest we put the onus on the wrong side. What we know best on this subject, I think, is that egalitarianism is *prima facie* incredible. We are entitled to reject it without owing any developed alternative. [54]

The reductive proposal offered in this paper is in this spirit. What we know best on the topic of the measure of knowledge is that it does not consist in cardinality and that it appears to consist in or be similarity. Whatever that is. Sometimes it is better to have no theory than to have a bad theory.

### **The Measure of Knowledge and the Aim of Belief**

The focus has been exclusively on a descriptive question in metaphysics (What is the correct measure on the domain of the true?). I have sought to avoid a distinct, normative question in epistemology (What is the good in the domain of belief?). But the descriptive issue has consequences for our understanding of epistemic normativity since whatever else may be epistemically good, knowing more and being less ignorant surely are. This is not the place to explore in detail the normative consequences of construing the measure of knowledge by appeal

to the similarity between a representation and what it represents. However, I do want to show how much recent discussion in epistemology of the aim of belief rests on an unstated assumption about the measure of knowledge, one that we have discovered in the course of this paper is false. And although I will not argue the point, I will suggest that construing the measure of knowledge by appeal to similarity preserves truth as a central aim of belief in an intuitively satisfying way.

A once-standard view of the aim of belief or inquiry is that belief or inquiry aims at truth. This is not a thesis about the motivations that guide individual cognizers in belief formation, but a claim about the constitutive or distinctly epistemic aim of belief. One set of challenges to this view centre on whether it can be right that truth is the only or principal thing of epistemic value. But a second set of challenges turn on whether it is even correct to think of truth in general, mere truth unqualified, as an aim of inquiry at all. The problem is that some truths seem to be not worth believing, or at least to be not much worth believing, even from an epistemic point of view. As Ernest Sosa aptly describes the problem:

Suppose you enter your dentist's waiting room and find all the magazines taken. Deprived of reading matter, you're sure to doze off, but you need no sleep. Are you then rationally bound to reach for the telephone book in pursuit of truth? Were you not to do so, you would forfeit a chance to pluck some desired goods within easy reach.

If random telephone numbers do not elicit a wide enough yawn, consider a randomly selected cubic foot of the Sahara. Here is a trove of facts, of the form *grain x is so many millimeters in direction D from grain y*, than which few can be of less interest. (2001, 49)

Examples abound: What is the 323rd entry in the Wichita, Kansas phone directory? (Goldman 1999, 88) How many threads are there in my carpet or shirt? (Lynch 2004, 55) Disjunctions where one disjunct is a proposition already believed, and “lots of other redundant 'garbage' of this sort”. (David 2005, 298) Trivial truths such as these are alleged to pose a problem for the view that inquiry aims at truth for a reason that is roughly captured by the following argument:

If inquiry aims at truth, it aims equally at every truth.  
Inquiry does not aim equally at every truth.  
Inquiry does not aim at truth.

There are alternative ways to develop the argument, but the core idea is that reflection on trivial truths teaches us that inquiry is not egalitarian toward truths, and this teaches us that mere truth is not what matters. This objection has been part of a broad turn away from truth as the goal of inquiry, one that is rightly taken to have far-reaching consequences for our understanding of epistemic normativity. Does inquiry aim not at truth but at 'understanding', where this need not, perhaps, even be factive? Is there a constitutive connection between inquiry, even as an idealized activity, and the satisfaction of curiosity? Might the difference between the trivial and the non-trivial, or at least some of it, be understood by appeal to a contribution to human flourishing? These and a hundred other flowers bloom.

Closer attention to the measure of knowledge shows that the argument moves too quickly. After all, if inquiry aims at truth, it does not follow from that alone that inquiry aims equally at every truth, in the sense that it is indiscriminate towards truths. The attraction to thinking so lies in the fact that all true propositions are equally true. But the implicit assumption is that any two truths contribute the same amount of truth to the balance sheet. In other words, the objection turns on the unstated assumption that if  $p$  is one truth and  $q$  another, then one knows as much by knowing  $p$  as by knowing  $q$ , and one is ignorant of as much by failing to know  $p$  as by failing to know  $q$ . But this assumption is false for a reason that should by now be familiar: cardinality does not exhaust the structure of what is true.

The trivial truths objection is correct to a point, however. Inquiry does not aim at every truth equally. But this is because it aims for truth, rather than grounds for thinking it doesn't.

Imagine someone arguing:

If gold mining aimed for gold, it would aim at every piece of gold equally – every piece of gold is equally gold, after all. But gold mining aims for flakes more than for dust, for nuggets more than for flakes, and for great veins more than anything else. So it does not aim for gold.

Here the metaphysical assumption is as obvious as it is obviously false. Gold mining is inegalitarian toward fragments of gold, that is true. But this does not show that gold mining does not aim at gold. Just the opposite: Gold mining aims for great veins *because* there's more gold there. Similarly, the reason inquiry does not aim (or aim very much) for trivial truths is because those truths are such that one does not increase one's knowledge, or decrease one's ignorance, very much by knowing them. The trivial truths objection fails, therefore, in an interesting way: properly understood, the fact that inquiry does not aim much for trivial truths manifests, rather than refutes, that inquiry aims at truth.

### Bibliography

- David Braddon-Mitchell and Frank Jackson. 2007. *Philosophy of Mind and Cognition: An Introduction*. Malden, MA: Blackwell.
- David Chalmers. 2002. The components of content. In *Philosophy of Mind: Classical and Contemporary Readings*. (ed. Chalmers), Oxford: OUP, pp. 608-633.
- Marian David. 2005. On 'Truth is Good'. *Philosophical Books*, 46:292–301.
- Donald Davidson. 2001a. Thought and talk. In *Inquiries into Truth and Interpretation* (pp. 155-171). Oxford: Oxford University Press.
- 2001b. A coherence theory of truth and knowledge. In *Subjective, Intersubjective, Objective* (pp. 137-153). Oxford: Oxford University Press.
- Jerry Fodor and Ernest Lepore. 1999. All at sea in semantic space: Churchland on meaning similarity. *Journal Of Philosophy* 96:381-403.
- Gottlob Frege. 1884. *The Foundations of Arithmetic*. Translated by J.L. Austin, 1980, Evanston, IL: Northwestern University Press.
- 1892. Sense and reference. In *Translations from the Philosophical Writings of Gottlob Frege*, translated by Peter Geach and Max Black, Oxford: Basil Blackwell, 1960.
- Risto Hilpinen. 1976. Approximate truth and truthlikeness, in *Formal Methods in the Methodology of the Empirical Sciences*, edited by M. Przelecki, K. Szaniawski and R. Wojcicki (eds.). Reidel. pp. 19-42.

- Peter Geach, 1980. *Reference and Generality*, 3rd ed. Cornell University Press.
- Alvin Goldman. 1999. *Knowledge in a Social World*. New York: Oxford University Press.
- David Lewis. 1983. New work for a theory of universals. *Australasian Journal of Philosophy*. 61:343-377.
- 1986a. *On the Plurality of Worlds*. Oxford: Blackwell.
- 1986b. Postscripts to “Counterfactual Dependence and Time's Arrow”, in *Philosophical Papers, Volume II*. New York: Oxford University Press.
- 1999. Reduction of mind. In *Papers in Metaphysics and Epistemology, Volume Two* (pp. 291-324). Cambridge, UK: Cambridge University Press.
- Michael Lynch. 2004. *True to Life: Why Truth Matters*. Cambridge, MA: The MIT Press.
- Alex Oliver. 1996. The metaphysics of properties. *Mind*. 105:417, pp. 1-80.
- W.V.O. Quine. 1960. *Word and Object*. Cambridge, MA: MIT Press.
- Ernest Sosa. 2001. For the love of truth? in *Virtue Epistemology: Essays on Epistemic Virtue and Responsibility*, edited by Abrol Fairweather and Linda Zagzebski, Oxford University Press.
- Robert Stalnaker. 2004. Lewis on intentionality. *Australasian Journal of Philosophy*, 82:1, pp. 199-212.
- Barry Stroud. 2000. The problem of the external world, in *Epistemology: An Anthology*, edited by J. Kim and E. Sosa, Blackwell. Originally printed in Ch. 1 *The Significance of Philosophical Skepticism*, Oxford: Clarendon Press, 1984.
- Michael Williams. 2001. *Problems of Knowledge*. Oxford: Oxford University Press.
- Neil Wilson. 1959. Substances without substrata, *Review of Metaphysics* 12, pp. 521-39.
- Ludwig Wittgenstein. 1969. *On Certainty*. Translated by G.E.M. Anscombe and D. Paul, Oxford: Blackwell.